Joint Congestion Control, Routing and MAC for Stability and Fairness in Wireless Networks

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***Abstract*— In this work, we describe and analyze a joint scheduling, routing and congestion control mechanism for wireless networks, that asymptotically guarantees stability of the buffers and fair allocation of the network resources. The queue-lengths serve as common information to different layers of the network protocol stack. Our main contribution is to prove the asymptotic optimality of a *primal-dual congestion controller*, which is known to model different versions of TCP well.**

Keywords: Primal-Dual Algorithm, Congestion Control, Lyapunov Stability Theory, Throughput-optimal Scheduling, Wireless Networks, Fair Resource Allocation, Nonlinear optimization.

I. Introduction

Consider a set of flows that share the resources of a fixed wireless network. Each flow is described by its source-destination node pair, with no a priori established routes. The limited power resources and interference amongst concurrent transmissions necessitate multihop transmission. The nodes that constitute the network must cooperate by forwarding each others' packets toward their destinations. Thus, each node may need to maintain buffers to hold packets of those flows other than its own. For such a system, we design a joint routing, MAC and congestion control algorithm that stabilizes the buffers, and drives the mean flow rates to a system-wide fair allocation point.

The question of designing stable scheduling algorithms for wireless networks was first addressed by Tassiulas and Ephremides[29] under the assumption that the incoming flows are *inelastic*, i.e., the flow rates are fixed as for voice or video traffic. They showed that scheduling transmissions as a function of the buffer occupancies (queue-lengths) stabilizes the queues. Tassiulas[27] extended this technique to derive a joint routing and scheduling algorithm that ensures the stability of the queues. These results showed that the queue-length-based resource allocation guarantees stability of the buffers as long as the arrival rates lie within the capacity (stability) region of the network. Subsequently, there has been a large body of work that extended the same idea to different scenarios and more general settings [30], [27], [28], [2], [24], [12], [22], [11]. However, these works do not consider the case of traffic whose rate can be adjusted online.

In the context of wireline networks, the idea of a distributed flow control based on a system-wide optimization problem was developed in [13], and followed by others in [19], [35], [15], [1], [31]; see [25] for a survey. In these works, the main contribution was the design of a distributed congestion control mechanism to drive the rates of *elastic* flows towards the system-wide optimum. In [36], [7], the authors use this idea to develop congestion control algorithms for wireless environments by reducing the available capacity region and converting the network into essentially a wireline network. The essential characteristics of wireless networks are not fully addressed there.

More recently, the problem of serving elastic traffic over wireless networks has been investigated in [23], [26], [16], [8], [10], [21], [17], [6]. Here, the queues and the wireless characteristics of the network are included in the system model. The main idea in these works has been to combine the results on scheduling inelastic traffic in wireless networks and distributed congestion control in wireline networks to design *joint scheduling-congestion control mechanisms* that guarantee optimal routes, stability and optimal rate allocation. These papers prove that a decentralized congestion controller at the transport layer working in conjunction with a queue-length-based scheduler at the medium access control (MAC) layer will asymptotically achieve buffer stability, optimal routing and fair rate allocation. Moreover, these layers are coupled through common queue-length information.

In [16], [10], [21], [26], the authors propose and study rate control algorithms that adapt the flow rates instantaneously as a function of the entry queue-lengths. The rate control mechanism studied in all of these works can be categorized as the *Dual Congestion Controller* since it can be interpreted as a gradient algorithm for the dual of an optimization problem. The intrinsic assumption of the dual congestion control mechanism is that the flow rates can be changed instantaneously in response to congestion feedback in the network. However, it is well known that adaptive window flow control mechanisms such as TCP respond to congestion feedback not instantaneously, but gradually. Such a response is desired by practitioners because the rate fluctuations are small. Thus, the study of another algorithm that modifies the flow rates gradually is important. To this end, we propose and study the so called *Primal-Dual Congestion Controller* in this work. Primal-dual algorithms are well known in the optimization literature and have been studied extensively in different contexts[1], [31], [25], [18]. Since the response of the primal-dual controller is more gradual compared to the dual controller, it is not immediately clear as to whether the buffer stability and rate convergence properties will be maintained. We note that the algorithm considered in [26] updates its rates

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somewhat differently than the algorithm in [10], [21], [16], [17], [6]. In [26], the users' data rates are still determined instantaneously as a function of the buffer occupancies and channel conditions, but an *average flow rate* is maintained for each user and used in the algorithm. On the other hand, in our work, we update the data rates to mimic the characteristics of widely-used versions of TCP [25]. Further, the proof technique used in [26] is quite different from ours, and our algorithm can be directly interpreted as a gradient algorithm for a primal optimization problem that is implemented at the sources and and a gradient algorithm for a dual optimization problem that is implemented at the nodes.

Here, it must be stressed that even though the congestion control is distributed, the scheduling is still assumed to be centralized in this work. In [17], [32], [4], [33], [5], the impact of decentralized implementations of the scheduler is studied. We note that the results of this work can be extended to distributed and asynchronous implementations for a special class of interference models using the approach in [4]. Finally, we note that a related, but different, problem has been considered in [34] where a distributed algorithm has been designed to route inelastic flows to minimize delay costs in a wireless network.

The rest of the paper is organized as follows: Section II describes the system model. In Section III, we state the objective of the resource allocation as an optimization problem and characterize the optimum point. Section IV introduces the queue-length-based resource allocation algorithm that is implemented at the MAC and network (routing) layers. We propose and study the primal-dual congestion controller (transport layer) in Section V. Various modifications and extensions to the system are described in Section VI. Finally, we give concluding remarks in Section VII.

II. System Model

We assume that the network is represented by a graph, $\mathcal{G} = (\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ is a set of nodes and $\mathcal{L}$ is a set of directed links. If a link (n, m) is in $\mathcal{L}$, then it is possible to send packets from node n to node m subject to the interference constraints to be described shortly. We let $\mu = \{\mu_l\}_{l\in\mathcal{L}}$ denote the vector of rates at which data can be transferred over each link $l \in \mathcal{L}$. We assume that there is an upper bound, $\hat{\eta} < \infty$, on each μ_l, which is a reasonable assumption for any practical system. We assume that zero is a feasible link rate for any link, independent of the link rates chosen for the other links in the network. Also, for ease of presentation we assume that there is no fading in the environment. We will discuss the extension of the model to include time-variations in Section VI.

We let $\hat{\Gamma}$ denote a bounded region in the $|\mathcal{L}|$ dimensional real space, representing the set of μ that can be achieved in a given time slot, i.e., it represents the interference constraint. In general, the set need not be convex. In fact, a typical case would be a discrete set of rates that can be achieved, and hence be non-convex. We let $\Gamma := \mathcal{CH}\{\hat{\Gamma}\}$ denote the convex hull of the set Γ. It is well known that by time-sharing between different rate vectors in $\hat{\Gamma}$, any point in Γ can be attained.

We use $\mathcal{F}$ to denote the set of flows that share the network resources. The routes of these flows are not specified a priori, but established by the back-pressure scheduling algorithm to be described in Section 2. We use $b(f)$ to denote the beginning node, and $e(f)$ to denote the end node of flow f. Figure 1 illustrates an example network with three flows passing through it.

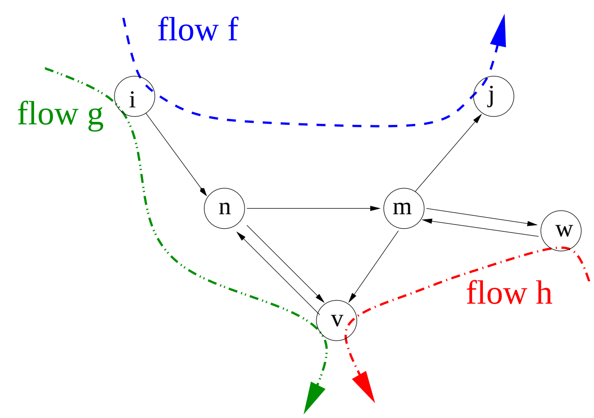

Fig. 1. An example network model with $b(f) = i$, $e(f) = j$, $b(g) = i$, $e(g) = v$, and $b(h) = w$, $e(h) = v$.

Associated with each flow f is a utility function $U_f(x_f)$, which is a function of the flow rate x_f. The utility function, denoted by $U_f(\cdot)$ for flow f, is assumed to satisfy the following conditions:

- $U_f(\cdot)$ is a twice differentiable, strictly concave, nondecreasing function of the mean flow rate, x_f.
- For every m and M satisfying $0 < m < M < \infty$, there exist constants $\tilde{c}$ and $\tilde{C}$ satisfying $0 < \tilde{c} < \tilde{C} < \infty$ such that

$$\tilde{c} \le -\frac{1}{U_f''(x)} \le \tilde{C} \qquad \forall x \in [m, M] \tag{1}$$

We note that these conditions are not especially restrictive and hold for the following class of utility functions.

$$U_f(x) = \beta_f \frac{x^{1-\alpha_f}}{(1-\alpha_f)} \qquad \forall\, \alpha_f > 0. \tag{2}$$

This class of utility functions is known to characterize a large class of fairness concepts including weighted-proportional and max-min fairness [20].

Next, we describe the capacity region of the network as in [22], [16].

Definition 1 (Capacity region): The *capacity region*, Λ, of the network contains the set of flow rates $\mathbf{x} \ge \mathbf{0}$ for which there exists a set $\left\{\mu_l^{(d)}\right\}_{l\in\mathcal{L}}^{d\in\mathcal{N}}$ that satisfies

(i) $\left[\sum_d \mu_l^{(d)}\right]_l \in \Gamma$, where $\mu_l^{(d)} \ge 0$ for all $l \in \mathcal{L}$, $d \in \mathcal{N}$.

(ii) For each $n \in \mathcal{N}$, and $d \ne n$,

$$\mu_{into(n)}^{(d)} + \sum_f x_f \mathcal{I}_{\{b(f)=n, e(f)=d\}} \le \mu_{out(n)}^{(d)},$$

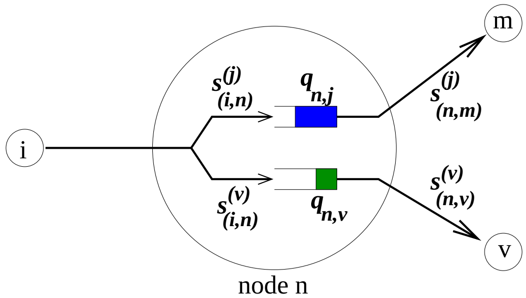

Fig. 2. Each node contains a queue for each destination node. This figure zooms into node n of Figure 1.

where

$$\mu^{(d)}_{into(n)} := \sum_{(k,n)\in\mathcal{L}} \mu^{(d)}_{(k,n)},$$
$$\mu^{(d)}_{out(n)} := \sum_{(n,m)\in\mathcal{L}} \mu^{(d)}_{(n,m)}.$$

Observe that $\mu^{(d)}_{into(n)}$ (or $\mu^{(d)}_{out(n)}$) denotes the potential number of packets that are destined for node d, incoming to (or outgoing from) node n. The condition (i) captures the interference constraints, while condition (ii) is the flow-conservation constraint that must hold at each node. $\diamond$

It is assumed that each node maintains a separate queue for those flows that have the same destination. We use $q_{n,d}[t]$ to denote the number of packets that are destined for node d, waiting for service at node n at time t. Figure 2 illustrates one such node. We set $q_{nn}[t] = 0$ for all $n \in \mathcal{N}$ and for all t. For each $n \in \mathcal{N}$, and $d \in \mathcal{N}\backslash\{n\}$, the evolution of $q_{n,d}$ is given by

$$\begin{aligned} q_{n,d}[t+1] = \quad & q_{n,d}[t] + \sum_f x_f[t]\,\mathcal{I}_{\{b(f)=n,e(f)=d\}} \\ & + s^{(d)}_{into(n)}[t] - s^{(d)}_{out(n)}[t], \end{aligned} \tag{3}$$

where we define $s^{(d)}_{into(n)}[t] := \sum_{(k,n)\in\mathcal{L}} s^{(d)}_{(k,n)}[t]$ and $s^{(d)}_{out(n)}[t] := \sum_{(n,m)\in\mathcal{L}} s^{(d)}_{(n,m)}[t]$, and $s^{(d)}_{(n,m)}[t]$ denotes the rate provided to d-destined packets over link (n,m) at slot t. Notice that, this is the actual amount of packets served over the link, not the potential amount denoted by $\mu^{(d)}_{(n,m)}[t]$. Clearly, we have $s_{(n,m)}[t] = \sum_d s^{(d)}_{(n,m)}[t]$. Also, $s^{(d)}_{(n,m)}[t] = \min(\mu^{(d)}_{(n,m)}[t], q_{n,d}[t])$ for all $(n,m)\in\mathcal{L},\ d \neq n$.

III. Problem Statement and Characterization of the Optimal Point

Our goal is to design a congestion control, scheduling mechanism such that the flow rate vector, $\mathbf{x}$, solves the following optimization problem:

$$\max_{\mathbf{x}\in\Lambda} \sum_{f\in\mathcal{F}} U_f(x_f). \tag{4}$$

We refer to (4) as the *primal problem*. Due to the strict concavity assumption of $U_f(\cdot)$ and the convexity of the capacity region Λ, there exists a unique optimizer of the primal problem, which we refer to as $\mathbf{x}^\star$. We call this the *fair rate allocation*.

One can use duality theory by defining $\lambda_{n,d}$ to be the Lagrange multiplier associated with the constraint

$$\mu^{(d)}_{into(n)} + \sum_f x_f \mathcal{I}_{\{b(f)=n,e(f)=d\}} \leq \mu^{(d)}_{out(n)}$$

to get the following dual function after algebraic manipulations (see the appendix for the details):

$$\begin{aligned} D(\lambda) \;=\; & \sum_{f\in\mathcal{F}} \max_{x_f\geq 0}\{U_f(x_f) - x_f\lambda_{b(f),e(f)}\} \\ & + \max_{\mu\in\Gamma} \sum_{(n,m)\in\mathcal{L}} \mu_{(n,m)} \max_{d\in\mathcal{N}}(\lambda_{n,d}-\lambda_{m,d}), \end{aligned} \tag{5}$$

where $\lambda_{d,d}$ is taken to be zero for all d. We note that we use μ to denote the link rate vector $\{\mu_{(n,m)}\}_{(n,m)\in\mathcal{L}}$, not $\{\mu^{(d)}_{(n,m)}\}^{d\in\mathcal{N}}_{(n,m)\in\mathcal{L}}$.

In the dual function, $\lambda_{n,d}$ can be interpreted as the price of transferring a unit amount of data from node n to node d. Thus, $\lambda_{b(f),e(f)}$ is nothing but the price of transferring a unit amount of data from the source of flow f to its destination. Such an approach was taken in [16] where it was shown that for this problem, the duality gap vanishes, and that there exists a nonempty set $\Psi^\star$ of optimal Lagrange multipliers that satisfy

$$\sum_{f\in\mathcal{F}} U_f(x^\star_f) = D(\lambda^\star), \text{ for all } \lambda^\star\in\Psi^\star.$$

The definition of the capacity region and the optimality conditions imply that for each $\lambda^\star$ there is an associated rate vector $\overset{\star}{\mu}\in\Gamma$ which satisfies:

(i) $\sum_d \overset{\star}{\mu}{}^{(d)}_{(n,m)} = \overset{\star}{\mu}_{(n,m)}$, for each $(n,m)\in\mathcal{L}$.

(ii) $\overset{\star}{\mu}{}^{(d)}_{into(n)} + \sum_f x^\star_f \mathcal{I}_{\{b(f)=n,e(f)=d\}} \leq \overset{\star}{\mu}{}^{(d)}_{out(n)}$, for all $n\in\mathcal{N}$ and $d\in\mathcal{N}\backslash\{n\}$.

(iii) $\overset{\star}{\mu} \in \arg\max_{\mu\in\Gamma} \sum_{(n,m)\in\mathcal{L}} \mu_{(n,m)} \max_d(\lambda^\star_{n,d}-\lambda^\star_{m,d})$.

(iv) $\lambda^\star_{n,d}(\overset{\star}{\mu}{}^{(d)}_{into(n)} + \sum_f x^\star_f \mathcal{I}_{\{b(f)=n,e(f)=d\}} - \overset{\star}{\mu}{}^{(d)}_{out(n)}) = 0$ for all $n\in\mathcal{N}$ and $d\in\mathcal{N}\backslash\{n\}$.

Using property (iv), and summing over all $n\in\mathcal{N}$ and $d\in\mathcal{N}$, we get

$$\begin{aligned} \sum_f x^\star_f \lambda^\star_{b(f),e(f)} &= \sum_{n\in\mathcal{N}}\sum_{d\in\mathcal{N}} \lambda^\star_{n,d}(\overset{\star}{\mu}{}^{(d)}_{out(n)} - \overset{\star}{\mu}{}^{(d)}_{into(n)}) \\ &= \sum_{n\in\mathcal{N}}\sum_{d\in\mathcal{N}} \lambda^\star_{n,d}\left(\sum_{k\neq n}\overset{\star}{\mu}{}^{(d)}_{(n,k)} - \sum_{j\neq n}\overset{\star}{\mu}{}^{(d)}_{(j,n)}\right) \\ &= \sum_{n\in\mathcal{N}}\sum_{d\in\mathcal{N}}\sum_{k\neq n} \lambda^\star_{n,d}\,\overset{\star}{\mu}{}^{(d)}_{(n,k)} \\ &\quad - \sum_{n\in\mathcal{N}}\sum_{d\in\mathcal{N}}\sum_{j\neq n} \lambda^\star_{n,d}\,\overset{\star}{\mu}{}^{(d)}_{(j,n)}\,. \end{aligned}$$

Next, we change the indices in the sums to make the link rates the same. This yields

$$
\begin{aligned}
&\sum_f x_f^\star \lambda^\star_{b(f),e(f)} \\
&\quad = \sum_{n\in\mathcal{N}} \sum_{m\neq n} \sum_{d\in\mathcal{N}} \lambda^\star_{n,d} \, \overset{\star}{\mu}{}^{(d)}_{(n,m)} \\
&\qquad - \sum_{m\in\mathcal{N}} \sum_{n\neq m} \sum_{d\in\mathcal{N}} \lambda^\star_{m,d} \, \overset{\star}{\mu}{}^{(d)}_{(n,m)} \\
&\quad = \sum_{(n,m)\in\mathcal{L}} \sum_{d\in\mathcal{N}} \overset{\star}{\mu}{}^{(d)}_{(n,m)} \, (\lambda^\star_{n,d} - \lambda^\star_{m,d}) \\
&\quad = \sum_{(n,m)\in\mathcal{L}} \overset{\star}{\mu}_{(n,m)} \, \max_d (\lambda^\star_{n,d} - \lambda^\star_{m,d}) \\
&\quad \geq \sum_{(n,m)\in\mathcal{L}} \mu_{(n,m)} \max_d (\lambda^\star_{n,d} - \lambda^\star_{m,d}), \qquad \forall \mu \in \Gamma, \quad (6)
\end{aligned}
$$

where the last inequality follows from the property (iii). This inequality will later be used in the proof of stability of the system.

IV. Scheduling Algorithm

In this paper, we use a queue-length-based scheduler known as the *back-pressure scheduler* introduced by Tassiulas and Ephremides [29]. This scheduler assigns a weight to each link that equals to the maximum differential backlog between the transmitting and receiving nodes, and then chooses link rates to maximize the sum of the product of link weights and link rates. The details of the scheduler is provided in the following definition.

Definition 2 (Back-pressure Scheduler): At slot t, for each $(n,m) \in \mathcal{L}$, we define the differential backlog for destination node d as

$$w_{(n,m),d}[t] := (q_{n,d}[t] - q_{m,d}[t]).$$

Also, we let

$$
\begin{aligned}
w_{(n,m)}[t] &= \max_d \left\{ w_{(n,m),d}[t] \right\} \qquad (7)\\
d_{(n,m)}[t] &= \arg\max_d \left\{ w_{(n,m),d}[t] \right\}.
\end{aligned}
$$

Choose the rate vector $\mu[t] \in \hat{\Gamma}$ that satisfies

$$\mu[t] \in \arg\max_{\{\eta\in\hat{\Gamma}\}} \sum_{\{(n,m)\in\mathcal{L}\}} \eta_{(n,m)} w_{(n,m)}[t], \qquad (8)$$

and then serve the queue holding packets destined for node $d_{(n,m)}[t]$ over link (n,m) at rate $\mu_{(n,m)}[t]$. That is, we set

$$\mu^{(d_{(n,m)}[t])}_{(n,m)}[t] = \mu_{(n,m)}[t].$$

The rest of the queues at node n are not served at slot t. $\diamond$

Such a resource allocation rule has been shown to achieve *throughput-optimality* [29], i.e., any arrival rate that can stabilize the network using any other resource allocation policy can be supported by this policy. Next, we list two other facts related to the back-pressure policy that will be used later.

Fact 1: The maximization in (8) can be performed over Γ instead of $\hat{\Gamma}$, because the optimal rate vector must always contain at least one element from $\hat{\Gamma}$. This follows from the linearity of the objective function and the fact that $\Gamma = \mathcal{CH}\{\hat{\Gamma}\}$.

Fact 2: Those flows that have $w_{l,d}[t] < 0$ will get $\mu_l^{(d)}[t] = 0$, because the objective of the optimization in (8) can only decrease by choosing $\mu_l^{(d)}[t] > 0$, if $w_{l,d}[t] < 0$.

V. Primal-Dual Congestion Controller

The function of the congestion control mechanism is to observe the congestion level of the network and respond to it by increasing/decreasing the data rate of the flows so that they evolve towards the fair allocation as described in Section III. In this paper, we propose a primal-dual congestion control mechanism that can be implemented in a decentralized fashion for each flow. In particular, the source node of each flow uses its local queue-length information as well as the utility function associated with that flow to update the flow rate in an iterative manner. This is similar, in principle, to window-based flow control mechanisms implemented in many versions of TCP, because for such mechanisms the flow rates are gradually increased or decreased depending on the congestion feedback from the network.

Definition 3 (Primal-Dual Congestion Controller): At the beginning of time slot t, each flow, say f, has access to the queue-length of its first node, i.e. $q_{b(f),e(f)}[t]$. The data rate $x_f[t]$ of flow f satisfies

$$x_f[t+1] = \left\{ x_f[t] + \alpha \left(K U_f' \left(x_f[t] \right) - q_{b(f),e(f)}[t] \right) \right\}_m^M,$$

where the notation $\{y\}_a^b$ denotes a projection of y to the closest point in the interval $[a,b]$. We assume that $m < \min_f x_f^*$ is a fixed positive number that can be arbitrarily small, and $M, K > 0$. $\diamond$

Later, we will see that the parameter K determines how closely the flow rates determined by the primal-dual congestion controller approximate the optimal rates $\mathbf{x}^*$.

In the following sections, we prove that this congestion control mechanism, when operated in conjunction with the back-pressure scheduler, achieves flow rates arbitrarily close to the fair allocation. To that end, we first study a heuristic fluid model, and then consider the original discrete-time system.

For purposes of simplicity, we will study the system under the following assumption in the main body of the paper.

Assumption 1: There is a flow between every source-destination (n,d) pair with $n \in \mathcal{N}, d \neq n$. $\diamond$

In Appendix B, we show that our results continue to hold without this assumption.

A. Convergence of the Primal-Dual Controller

In this section, we first introduce a heuristic fluid model of the joint scheduler-congestion control mechanism, and prove its stability and convergence properties, and then show the convergence properties of the discrete-time primal-dual algorithm using the results of the fluid model.

1) Analysis of a continuous-time fluid model: We first present LaSalle's invariance principle, used to determine the stability of differential equations [14], which will be useful for subsequent analysis.

Theorem 1 (LaSalle's Invariance Principle): Consider the differential equation: $\dot{\mathbf{y}}(t) = f(\mathbf{y}(t))$. Let $Y : D \to \mathcal{R}$ be a radially unbounded[1], continuously differentiable, positive definite [2] function such that $\dot{Y}(\mathbf{z}) \leq 0$ for all $\mathbf{z} \in D$. Let $\mathcal{E}$ be the set of points in D where $\dot{Y}(\mathbf{z}) = 0$. Let $\mathcal{M}$ be the largest invariant set[3] in $\mathcal{E}$. Then, every solution starting in D approaches $\mathcal{M}$ as $t \to \infty$. $\diamond$

Now, we present the heuristic fluid model of the primal-dual algorithm. We assume that time is continuous and the evolution of each queue is governed by the differential equation: for each $n \in \mathcal{N}$, and $d \in \mathcal{N} \backslash \{n\}$,

$$\dot{q}_{n,d}(t) = \left(\sum_f x_f(t)\, \mathcal{I}_{b(f)=n,e(f)=d} + \mu^{(d)}_{into(n)}(t) - \mu^{(d)}_{out(n)}(t)\right)_{q_{n,d}(t)\geq 0}, \tag{9}$$

where $(y)_{z\geq a}$ is equal to y when $z > a$ and is equal to $\max(y, 0)$ when $z = a$. Here, (t) is used instead of $[t]$ to signify that we are working in continuous-time. The back-pressure algorithm computes the link schedules and rates at every instant of time as described in Section 2. Finally, the congestion controller is assumed to determine the instantaneous flow rates such that, for all $f \in \mathcal{F}$,

$$\dot{x}_f(t) = \alpha \left(KU'_f\left(x_f(t)\right) - q_{b(f),e(f)}(t)\right)_{x_f(t)\geq m}. \tag{10}$$

Then, the following global asymptotic stability result holds.

Theorem 2: Starting from any initial condition $(\mathbf{x}(0), \mathbf{q}(0))$, the state of the system $(\mathbf{x}(t), \mathbf{q}(t))$ converges to $(\mathbf{x}^\star, K\lambda^\star)$ as $t \to \infty$, where $\lambda^\star := \{\lambda^\star_{n,d}\}$ is given by $\lambda^\star_{b(f),e(f)} = U'_f(x^\star_f)$ for each f.

Proof: The proof is based on LaSalle's invariance principle. As in the case of the primal-dual congestion controller for the Internet [25], we start with the following Lyapunov function

$$Y(\mathbf{x}, \mathbf{q}; \lambda^\star) := \sum_{f\in\mathcal{F}} \frac{(x_f - x^\star_f)^2}{2\alpha K} + \frac{K}{2} \sum_{n\in\mathcal{N}} \sum_{d\in\mathcal{N}} \left(\frac{q_{n,d}}{K} - \lambda^\star_{n,d}\right)^2. \tag{11}$$

It is easy to see that this is a radially unbounded function. Next, we study time-derivative of this function.

$$\dot{Y}(\mathbf{x}(t), \mathbf{q}(t); \lambda^\star)$$

$$= \sum_{f\in\mathcal{F}} (x_f(t) - x^\star_f) \left(U'_f\left(x_f(t)\right) - \frac{q_{b(f),e(f)}(t)}{K}\right)_{x_f(t)\geq m} \tag{12}$$

$$+ \sum_{n,d} \left(\frac{q_{n,d}(t)}{K} - \lambda^\star_{n,d}\right) \left(\sum_f x_f(t)\, \mathcal{I}_{b(f)=n,e(f)=d} + \mu^{(d)}_{into(n)}(t) - \mu^{(d)}_{out(n)}(t)\right)_{q_{n,d}(t)\geq 0} \tag{13}$$

$$\leq \sum_{f\in\mathcal{F}} (x_f(t) - x^\star_f) \left(U'_f\left(x_f(t)\right) - \frac{q_{b(f),e(f)}(t)}{K}\right) \tag{14}$$

$$+ \sum_{n,d} \left(\frac{q_{n,d}(t)}{K} - \lambda^\star_{n,d}\right) \left(\sum_f x_f(t)\, \mathcal{I}_{b(f)=n,e(f)=d} + \mu^{(d)}_{into(n)}(t) - \mu^{(d)}_{out(n)}(t)\right), \tag{15}$$

where (14) follows from (12) due to the assumption that $m < x^\star_f$, $\forall f$. Similarly, (15) follows from (13) when we note that removing the lower bound on $q_{n,d}(t)$ can only increase the sum since $\lambda^\star_{n,d} \geq 0$ by definition.

Let us add and subtract $U'_f(x^\star_f) = \lambda^\star_{b(f),e(f)}$, and $\sum_f x^\star_f \mathcal{I}_{\{b(f)=n,e(f)=d\}}$ to (14) and (15), respectively, and rearrange terms to yield $\dot{Y}(\mathbf{x}(t), \mathbf{q}(t); \lambda^\star)$

$$\leq \sum_f (x_f(t) - x^\star_f)(U'_f(x_f(t)) - U'_f(x^\star_f)) \tag{16}$$

$$+ \sum_f (x_f(t) - x^\star_f)\left(\lambda^\star_{b(f),e(f)} - \frac{q_{b(f),e(f)}(t)}{K}\right) \tag{17}$$

$$+ \sum_f \left(\frac{q_{b(f),e(f)}(t)}{K} - \lambda^\star_{b(f),e(f)}\right)(x_f(t) - x^\star_f) \tag{18}$$

$$+ \sum_{n,d} \lambda^\star_{n,d} \left(\mu^{(d)}_{out(n)}(t) - \mu^{(d)}_{into(n)}(t) - \sum_f x^\star_f \mathcal{I}_{\{b(f)=n,e(f)=d\}}\right) \tag{19}$$

$$+ \sum_{n,d} \frac{q_{n,d}(t)}{K} \left(\sum_f x^\star_f \mathcal{I}_{\{b(f)=n,e(f)=d\}} + \mu^{(d)}_{into(n)}(t) - \mu^{(d)}_{out(n)}(t)\right). \tag{20}$$

Notice that(17) and (18) cancel each other. The strict concavity of $U_f(\cdot)$ implies that $(16) \leq 0$ for all $\mathbf{x}(t)$ with strict inequality whenever $\mathbf{x}(t) \neq \mathbf{x}^\star$. Next, we study the (19) and (20) separately to argue that they are both upper-bounded by zero.

We start with (19). Notice that we can write

$$\sum_{n,d} \lambda^\star_{n,d} \left(\mu^{(d)}_{out(n)}(t) - \mu^{(d)}_{into(n)}(t)\right)$$

$$= \sum_{(n,m)\in\mathcal{L}} \sum_{d\neq n} \mu^{(d)}_{(n,m)}(t)(\lambda^\star_{n,d} - \lambda^\star_{m,d})$$

$$\leq \sum_f x^\star_f \lambda^\star_{b(f),e(f)},$$

[1]Function $F(\mathbf{z})$ is called *radially unbounded* if $\lim_{\|\mathbf{z}\|\to\infty} F(\mathbf{z}) = \infty$.

[2]Y is positive definite if $Y(\mathbf{z}^\star) = \mathbf{0}$ for some $\mathbf{z}^\star$, and $Y(\mathbf{z}) > 0$ for all $\mathbf{z} \neq \mathbf{z}^\star$.

[3]A set $\mathcal{M}$ is said to be an *invariant set* if $\mathbf{z}(0) \in \mathcal{M}$ implies that $\mathbf{z}(t) \in \mathcal{M}$ for all $t \geq 0$.

where the equality follows from a change in the order of summation, and the inequality is due to (6). Therefore, we have $(19) \leq 0$.

Next, we consider the expression (20). Recall the flow-balance condition for destination d at node n introduced in Section III:

$$\sum_{f:b(f)=n,e(f)=d} x_f^\star \quad \leq \quad \overset{\star}{\mu}{}^{(d)}_{out(n)} - \overset{\star}{\mu}{}^{(d)}_{into(n)}, \qquad \forall n, d.$$

Next, we multiply both sides of this expression by $q_{n,d}(t)$ and sum over all n, d to get:

$$\begin{aligned}
&\sum_{n,d} \sum_{f:b(f)=n,e(f)=d} x_f^\star q_{n,d}(t) \\
&\quad \leq \quad \sum_{n,d} (\overset{\star}{\mu}{}^{(d)}_{out(n)} - \overset{\star}{\mu}{}^{(d)}_{into(n)}) q_{n,d}(t) \\
&\quad = \quad \sum_{(n,m)\in\mathcal{L}} \sum_{d} \overset{\star}{\mu}{}^{(d)}_{(n,m)} \; (q_{n,d}(t) - q_{m,d}(t)) \\
&\quad \overset{(a)}{\leq} \quad \sum_{(n,m)\in\mathcal{L}} \sum_{d} \mu^{(d)}_{(n,m)}(t)(q_{n,d}(t) - q_{m,d}(t)), \quad (21)
\end{aligned}$$

where the inequality (a) holds due to (8). This shows that $(20) \leq 0$.

Note that since $(17) + (18) = 0$, we have $\dot{Y}(\mathbf{x}(t), \mathbf{q}(t); \lambda^\star) \leq (16) + (19) + (20)$, and we have just shown that $(16) \leq 0$, $(19) \leq 0$, and $(20) \leq 0$. This implies that $\dot{Y}(\mathbf{x}(t), \mathbf{q}(t); \lambda^\star) \leq 0$, and further it also implies that

$$\mathcal{E} := \{(\mathbf{x}, \mathbf{q}) \; : \; \dot{Y}(\mathbf{x}, \mathbf{q}; \lambda^\star) = 0\}$$

is contained in the set

$$\mathcal{S} \quad := \quad \{(\mathbf{x}, \mathbf{q}) : \quad (16) = (19) = (20) = 0\}.$$

Let $\mathcal{M}$ be the largest invariant set of the primal-dual algorithm contained in $\mathcal{E}$. By LaSalle's invariance principle $(\mathbf{x}(t), \mathbf{q}(t))$ converges to the set $\mathcal{M}$ as $t \to \infty$. Since $\mathcal{M} \subset \mathcal{E} \subset \mathcal{S}$, as $t \to \infty$, the pair $(\mathbf{x}(t), \mathbf{q}(t))$ must also satisfy $(16) = 0$. Then, strict concavity of the utility functions imply that $\lim_{t\to\infty} x_f(t) = x_f^\star$ for each flow f.

Further, since $\lambda^\star_{b(f),e(f)} = U'_f(x_f^\star)$, the set $\{\lambda^\star_{n,d}\}$ is uniquely determined. For any $(\mathbf{x}, \mathbf{q}) \in \mathcal{M}$, if $\frac{q_{n,d}(t)}{K} \neq \lambda^\star_{n,d}$, then $\dot{x}_f(t) \neq 0$, and hence $x_f(t)$ will not stay at $x_f^\star$. Thus, a trajectory starting at such a $(\mathbf{x}, \mathbf{q})$ cannot stay in $\mathcal{S}$, and since $\mathcal{M}$ is the largest invariant set in $\mathcal{E} \subset \mathcal{S}$, such an $(\mathbf{x}, \mathbf{q}) \notin \mathcal{M}$. This implies that if $(\mathbf{x}, \mathbf{q}) \in \mathcal{M}$, then $\mathbf{x} = \mathbf{x}^\star$, and $\mathbf{q} = K\lambda^\star$. ∎

2) Analysis of the discrete-time model: Recall that the evolution of the flow rates and queue lengths are given by Definition 3 and (3), respectively. Throughout, we assume that $x_f^\star > m, \forall f$, which is a reasonable assumption given that we are free to choose m as small as necessary to satisfy it.

The following lemma provides a relationship between potential service rate μ and the actual service rate $\mathbf{s}$, which will be used in the proof of the subsequent theorem.

Lemma 1: The following relationship holds for any $\mathbf{q}[t]$ and some $B < \infty$:

$$\begin{aligned}
&\sum_{(n,m)\in\mathcal{L}} \sum_{d\neq n} s^{(d)}_{(n,m)}[t](q_{n,d}[t] - q_{m,d}[t]) \\
&\quad \geq \quad \sum_{(n,m)\in\mathcal{L}} \sum_{d\neq n} \mu^{(d)}_{(n,m)}[t](q_{n,d}[t] - q_{m,d}[t]) - B
\end{aligned}$$

Proof: We prove this lemma by considering three cases.
CASE 1: $q_{n,d}[t] < q_{m,d}[t]$: then, due to Fact 2, we have $\mu^{(d)}_{(n,m)}[t] = 0$ and subsequently, we must have $s^{(d)}_{(n,m)}[t] = 0$.
CASE 2: $q_{n,d}[t] \geq q_{m,d}[t]$ and $q_{n,d}[t] \geq \hat{\eta}$: then there can be no unused service since $\mu^{(d)}_{(n,m)}[t] < \hat{\eta}$ by assumption. Thus, we have $s^{(d)}_{(n,m)}[t] = \mu^{(d)}_{(n,m)}[t]$.
CASE 3: $\hat{\eta} > q_{n,d}[t] \geq q_{m,d}[t]$: then we have $s^{(d)}_{(n,m)}[t] < \hat{\eta}$ and $\mu^{(d)}_{(n,m)}[t](q_{n,d}[t] - q_{m,d}[t]) \leq \hat{\eta}^2$. Thus, in this case,

$$\begin{aligned}
&(s^{(d)}_{(n,m)}[t] - \mu^{(d)}_{(n,m)}[t])(q_{n,d}[t] - q_{m,d}[t]) \\
&\qquad \geq \quad -\mu^{(d)}_{(n,m)}[t](q_{n,d}[t] - q_{m,d}[t]) \\
&\qquad \geq \quad -\mu^{(d)}_{(n,m)}[t] q_{n,d}[t] \\
&\qquad \geq \quad -\hat{\eta}^2.
\end{aligned}$$

Combining the three cases, we have

$$(s^{(d)}_{(n,m)}[t] - \mu^{(d)}_{(n,m)}[t])(q_{n,d}[t] - q_{m,d}[t]) \geq -\hat{\eta}^2 L_3,$$

where L_3 is the number of indices (n, m, d) which satisfy the conditions of Case 3. Clearly, $L_3 \leq |\mathcal{L}||\mathcal{N}|$, and thus, choosing $B := |\mathcal{L}||\mathcal{N}|\hat{\eta}^2$ gives the desired result. ∎

The next proposition establishes the asymptotic boundedness of the queue-lengths, and hence the stability of the system.

Proposition 1: There exists a constant $c(\alpha, K, \tau) < \infty$ that depends on α and K and a free parameter $\tau \in \mathbf{Z}_+$ such that

$$\limsup_{t\to\infty} \sum_{n,d} q^2_{n,d}[t] \leq c(\alpha, K, \tau).$$

Further, $c(\alpha, K, \tau)$ is such that, when α is chosen to be $1/K^2$ and τ is chosen to be K, then $c(\alpha, K, \tau)$ is of the order of K^2, i.e., $c(1/K^2, K, K)/K^2$ tends to a constant as $K \to \infty$.

Proof: Let us consider the Lyapunov function

$$L(\mathbf{q}) = \frac{1}{2} \sum_{n\in\mathcal{N}} \sum_{d\in\mathcal{N}} q^2_{n,d},$$

and study its drift:

$$\begin{aligned}
\Delta L_t(\mathbf{q}) \quad &:= \quad L(\mathbf{q}[t+1]) - L(\mathbf{q}[t]) \\
&\leq \quad B_1 + \sum_{n,d} q_{n,d}[t] \Bigg(\sum_f x_f[t] \mathcal{I}_{\{b(f)=n, e(f)=d\}} \\
&\qquad\qquad + s^{(d)}_{into(n)}[t] - s^{(d)}_{out(n)}[t] \Bigg),
\end{aligned}$$

for some $B_1 < \infty$ that is a function of M and the maximum link rate $\hat{\eta}$. Thus, we have

$$\begin{aligned}\Delta L_t(\mathbf{q}) &\leq B_1 + \sum_f q_{b(f),e(f)}[t]x_f[t] \\ &\quad - \sum_{n,d} q_{n,d}(s^{(d)}_{out(n)}[t] - s^{(d)}_{into(n)}[t]) \\ &= B_1 + \sum_f q_{b(f),e(f)}[t]x_f[t] \\ &\quad - \sum_{(n,m)}\sum_d s^{(d)}_{(n,m)}[t](q_{n,d}[t] - q_{m,d}[t]) \\ &\overset{(a)}{\leq} B_1 + B + \sum_f q_{b(f),e(f)}[t]x_f[t] \\ &\quad - \sum_{(n,m)}\sum_d \mu^{(d)}_{(n,m)}[t](q_{n,d}[t] - q_{m,d}[t]) \\ &= B_1 + B + \sum_f q_{b(f),e(f)}[t](x_f[t] - x^\star_f) \\ &\quad + \sum_f q_{b(f),e(f)}[t]x^\star_f \\ &\quad - \sum_{(n,m)}\sum_d \mu^{(d)}_{(n,m)}[t](q_{n,d}[t] - q_{m,d}[t])\end{aligned}$$

where the inequality (a) follows from Lemma 1, and the last equality is obtained by adding and subtracting the term with $\mathbf{x}^\star$. We have already shown in the derivation of (21) that

$$\sum_f q_{b(f),e(f)}[t]x^\star_f \leq \sum_{(n,m)}\sum_d \mu^{(d)}_{(n,m)}[t](q_{n,d}[t] - q_{m,d}[t]).$$

Hence, we have

$$\Delta L_t(\mathbf{q}) \leq B_1 + B + \sum_f q_{b(f),e(f)}[t](x_f[t] - x^\star_f). \quad (22)$$

To complete the proof, we will show that when the queue lengths are sufficiently large, the link rates are small making the above drift negative. To that end, we first define

$$r := \hat{\eta} + \frac{K}{\tau}\max_f U'_f(m) + \frac{M}{\alpha\tau^2},$$

where τ is an integer to be determined. Note that the total arrival rate into a queue can be bounded by some $A_{max} < \infty$ since the arrival rate of each flow is bounded by M, and the link rates are bounded by $\hat{\eta}$. Thus, $q_{n,d}[t] \leq q_{n,d}[t-1] + A_{max}$.

Then, we claim that if $q_{b(f),e(f)}[t] > \tau(r + A_{max})$ for any flow f and any $t \geq \tau$, then we must have $x_f[t] = m$. To see this, note that $q_{b(f),e(f)}[t] > \tau(r + A_{max})$ implies that $q_{b(f),e(f)}[t-\tau] > \tau r$ since in each slot, the queue-length cannot increase by more than A_{max} packets. Therefore, for each time slot $i \in \{t - \tau + 1, \cdots, t\}$ in which the rate $x_f[i]$ of Flow-i is larger than m, we have

$$\begin{aligned}&x_f[i] - x_f[i-1] \\ &\quad\leq \alpha(KU'_f(x_f[i-1]) - q_{b(f),e(f)}[i-1]) \\ &\quad\overset{(a)}{\leq} \alpha KU'_f(m) - \alpha\tau(r - \hat{\eta}) \\ &\quad\leq \alpha KU'_f(m) - \alpha\tau\left(\frac{K}{\tau}\max_f U'_f(m) + \frac{M}{\alpha\tau^2}\right) \\ &\quad\leq -\frac{M}{\tau},\end{aligned}$$

where inequality (a) follows from the fact that $x_f[\cdot] \geq m$ and that $U_f(\cdot)$ is a concave function. This further implies that, for each $i \in \{t - \tau + 1, \cdots, t\}$, $x_f[i]$ will keep decreasing by at least M/τ in each slot until it hits its minimum level of m, and stay at that level until time slot t. Thus, even if $x_f[t - \tau] = M$, at the end of the subsequent τ slots, the flow rate will certainly decrease to $x_f[t] = m$, which proves our claim. Building on this claim, we let

$$g(\alpha, K, \tau) := \tau(r + A_{max}) = \frac{M}{\alpha\tau} + (A_{max} + \hat{\eta})\tau + K\max_f U'_f(m).$$

Then, we have

$$\begin{aligned}&\textstyle\sum_f q_{b(f),e(f)}[t](x_f[t] - x^\star_f) \\ &\quad= \sum_{q_{b(f),e(f)}[t] \geq g(\alpha,K,\tau)} q_{b(f),e(f)}[t](x_f[t] - x^\star_f) \\ &\quad\quad+ \sum_{q_{b(f),e(f)}[t] < g(\alpha,K,\tau)} q_{b(f),e(f)}[t](x_f[t] - x^\star_f) \\ &\quad\leq \sum_{q_{b(f),e(f)}[t] \geq g(\alpha,K,\tau)} q_{b(f),e(f)}[t](m - x^\star_f) \\ &\quad\quad+ |\mathcal{F}|g(\alpha, K, \tau)(M - m),\end{aligned}$$

where in the last step, we used the fact that $x_f[t] \in [m, M]$, and $x^\star_f \geq m$, for all f. To bound the remaining sum, note that we have $m - x^\star_f \leq -\delta$, for some $\delta > 0$, which follows from our assumption of $x^\star_f > m$ for all f. Then, we can write

$$\begin{aligned}&\textstyle\sum_f q_{b(f),e(f)}[t](x_f[t] - x^\star_f) \\ &\leq -\delta\left[\sum_f q_{b(f),e(f)}[t] - \sum_{f: q_{b(f),e(f)}[t] < g(\alpha,K,\tau)} q_{b(f),e(f)}[t]\right] \\ &\quad + |\mathcal{F}|g(\alpha, K, \tau)(M - m) \\ &\leq -\delta\sum_f q_{b(f),e(f)}[t] + B_2(\alpha, K, \tau),\end{aligned}$$

where we define $B_2(\alpha, K, \tau) = |\mathcal{F}|g(\alpha, K, \tau)(M - m + \delta)$. Using this bound in (22) and noting that, by our assumption[4], there exists a flow between all source destination pairs, we can write

$$\Delta L_t(\mathbf{q}) \leq -\delta\sum_{n,d} q_{n,d}[t] + B_2(\alpha, K, \tau) + B_1 + B.$$

Thus, if $\sum_{n,d} q_{n,d}[t] \geq (B + B_1 + B_2(\alpha, K, \tau) + \epsilon)/\delta$, then $\Delta L_t(\mathbf{q}) \leq -\epsilon$. Also note that, since

$$\sum_{n,d} q_{n,d}[t] \geq \sqrt{\sum_{n,d} q^2_{n,d}[t]} = \sqrt{2L(\mathbf{q}[t])},$$

if $L(\mathbf{q}[t]) \geq \frac{1}{2}\left[(B + B_1 + B_2(\alpha, K, \tau) + \epsilon)/\delta\right]^2$, then, $\Delta L_t(\mathbf{q}) \leq -\epsilon$. Further, $\Delta L_t(\mathbf{q}) \leq (B + B_1 + B_2(\alpha, K, \tau))$, otherwise. These facts imply that, as $t \to \infty$,

$$L(\mathbf{q}[t]) \leq \left(\frac{B + B_1 + B_2(\alpha, K, \tau) + \epsilon}{\delta\sqrt{2}}\right)^2 + (B + B_1 + B_2(\alpha, K, \tau)).$$

Defining the right-hand side of the above inequality to be $c(\alpha, K, \tau)$, and observing that it is growing as K^2 when $\alpha = 1/K^2$ and $\tau = K$. ∎

[4]This assumption is removed in the appendix.

Next, we state the main theorem which shows that the average rate obtained by each user can be made arbitrarily close to its *fair* share (as defined by the resource allocation problem (4)) by letting K become large and choosing $\alpha = 1/K^2$. If the step-size α is selected as $1/K^2$ and the free parameter τ of Proposition 1 is selected as K, then $c(\alpha, K, K) = O(K^2)$ from Proposition 1, and the sum of the queue lengths in the network (also known as backlog) is upper-bounded by $O(K)$. Thus, assuming that the upper bound is a reasonable estimate of the backlog, there exists a tradeoff between backlog and fairness, which can be controlled through the choice of K. If K is large, the asymptotic rate allocation is close to the fair allocation but at the cost of larger backlog.

Theorem 3: If $\alpha = 1/K^2$, then, for some finite $\tilde{B} \in (0, \infty)$, we have the following result: for all $f \in \mathcal{F}$,

$$\begin{aligned} x_f^\star - \frac{\tilde{B}}{\sqrt{K}} &\leq \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} x_f[t] \\ &\leq \limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} x_f[t] \leq x_f^\star + \frac{\tilde{B}}{\sqrt{K}}. \end{aligned}$$

Proof: We study the drift of the Lyapunov function $Y(\cdot)$ given in (11):

$$\Delta Y_t(\mathbf{x}, \mathbf{q}; \lambda^\star) := Y(\mathbf{x}[t+1], \mathbf{q}[t+1]; \lambda^\star) - Y(\mathbf{x}[t], \mathbf{q}[t]; \lambda^\star),$$

which can be upper-bounded by using the same line of reasoning we followed in the proof of Theorem 2, i.e., we handle the boundary constraints of the rates and queue-lengths, add and subtract $U'_f(x_f^\star) = \lambda^\star_{b(f),e(f)}$, and $\sum_f x_f^\star \mathcal{I}_{\{b(f)=n,e(f)=d\}}$, and rearrange terms to get

$$\begin{aligned} &\Delta Y_t(\mathbf{x}, \mathbf{q}; \lambda^\star) \\ &\leq \sum_f (U'_f(x_f[t]) - U'_f(x_f^\star))(x_f[t] - x_f^\star) \qquad (23) \\ &\quad + \sum_f (x_f[t] - x_f^\star)\Big(\lambda^\star_{b(f),e(f)} - \frac{q_{b(f),e(f)}[t]}{K}\Big) \qquad (24) \\ &\quad + \sum_f \frac{\alpha}{2K}\left(KU'_f(x_f[t]) - q_{b(f),e(f)}[t]\right)^2 \qquad (25) \\ &\quad + \sum_f \Big(\frac{q_{b(f),e(f)}[t]}{K} - \lambda^\star_{b(f),e(f)}\Big)(x_f[t] - x_f^\star) \qquad (26) \\ &\quad + \sum_{n,d} \lambda^\star_{n,d}\Big(s^{(d)}_{out(n)}[t] - s^{(d)}_{into(n)}[t] \\ &\qquad - \sum_f x_f^\star \mathcal{I}_{\{b(f)=n,e(f)=d\}}\Big) \qquad (27) \\ &\quad + \sum_{n,d} \frac{q_{n,d}[t]}{K}\Big(\sum_f x_f^\star \mathcal{I}_{\{b(f)=n,e(f)=d\}} \\ &\qquad + s^{(d)}_{into(n)}[t] - s^{(d)}_{out(n)}[t]\Big) \qquad (28) \\ &\quad + \frac{1}{2K}\sum_{n,d}\Big(\sum_f x_f \mathcal{I}_{\{b(f)=n,e(f)=d\}} \\ &\qquad + s^{(d)}_{into(n)}[t] - s^{(d)}_{out(n)}[t]\Big)^2. \qquad (29) \end{aligned}$$

We claim that $(23) \leq -\tilde{C}\|\mathbf{x}[t] - \mathbf{x}^\star\|^2$, where $\tilde{C}$ is a positive constant that is independent of K. To see this, first note that the strict concavity assumption of the utility functions allows us to write

$$\begin{aligned} &(U'_f(x_f[t]) - U'_f(x_f^\star))(x_f[t] - x_f^\star) \\ &\quad = -\left|U'_f(x_f[t]) - U'_f(x_f^\star)\right|\left|x_f[t] - x_f^\star\right|, \end{aligned} \qquad (30)$$

for each $f \in \mathcal{F}$. Also, by the mean-value theorem, we can find some $y_f[t]$ between $x_f[t]$ and $x_f^\star$ for which $U'_f(x_f[t]) - U'_f(x_f^\star) = (x_f[t] - x_f^\star)U''_f(y_f[t])$. It follows from (1) that there exists some $\tilde{C} > 0$ such that $\left|U'_f(x_f[t]) - U'_f(x_f^\star)\right| \geq \tilde{C}|x_f[t] - x_f^\star|$, which can be substituted into (30) to prove the claim.

Observe that the terms (24) and (26) cancel each other. Also, notice that (27) and (28) are almost the same as (19) and (20), respectively, except for the fact that actual service rates appear in them instead of potential service rates. First, note that $(27) \leq 0$ since it can be written as

$$\begin{aligned} &\sum_{n,m}\sum_d s^d_{n,m}(\lambda^*_{n,d} - \lambda^*_{m,d}) - \sum_f x_f^* \lambda^*_{b(f),e(f)} \\ &\leq \sum_{n,m}\sum_d \overset{\star}{\mu}{}^d_{(n,m)}(\lambda^*_{n,d} - \lambda^*_{m,d}) - \sum_f x_f^*\lambda^*_{b(f),e(f)} \leq 0. \end{aligned}$$

To see that (28) is bounded, we argue as follows:

$$\begin{aligned} &(28) \\ &\leq \frac{B}{K} + \sum_{n,d}\frac{q_{n,d}}{K} \\ &\qquad \times \Big(\sum_{f: b(f)=n,e(f)=d} x_f^\star - \mu^{(d)}_{into(n)}[t] - \mu^{(d)}_{out(n)}[t]\Big) \\ &= \frac{B}{K} + (20) \leq \frac{B}{K}, \end{aligned}$$

where the first inequality follows from Lemma 1 and $(20) \leq 0$ as in the proof of Theorem 2.

Since the link rates and the flow rates are upper-bounded, we can find some $B_3 < \infty$ such that (29)$< B_3/K$. Thus, we have

$$\begin{aligned} \Delta Y_t(\mathbf{x}, \mathbf{q}; \lambda^\star) &\leq \frac{B + B_3}{K} - \tilde{C}\|\mathbf{x}[t] - \mathbf{x}^\star\|^2 \\ &\quad + \sum_f \frac{\alpha}{2K}\left(KU'_f(x_f[t]) - q_{b(f),e(f)}[t]\right)^2. \end{aligned}$$

If we write this drift expression for $t = 0, \cdots, T-1$ and add both sides of the inequalities, then we get

$$\begin{aligned} &Y(\mathbf{x}[T], \mathbf{q}[T]; \lambda^\star) - Y(\mathbf{x}[0], \mathbf{q}[0]; \lambda^\star) \\ &\quad \leq T\frac{B + B_3}{K} - \tilde{C}\sum_{t=0}^{T-1}\|\mathbf{x}[t] - \mathbf{x}^\star\|^2 \\ &\qquad + \sum_{t=0}^{T-1}\sum_f \frac{\alpha}{2K}\left(KU'_f(x_f[t]) - q_{b(f),e(f)}[t]\right)^2. \end{aligned}$$

By rearranging terms, noting that $Y(\cdot)$ is a non-negative quantity, dividing both sides by T, and taking the limit as T

goes to infinity yields

$$\limsup_{T\to\infty} \frac{\tilde{C}}{T}\sum_{t=0}^{T-1}\|\mathbf{x}[t]-\mathbf{x}^\star\|^2$$
$$\leq \limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1}\sum_{f\in\mathcal{F}}\frac{\alpha}{2K}\left(KU_f'(x_f[t])-q_{b(f),e(f)}[t]\right)^2 + \frac{B+B_3}{K}. \quad (31)$$

Thus, the proof will be complete once we show that, when $\alpha = 1/K^2$, we have

$$\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1}\sum_{f\in\mathcal{F}}\frac{\alpha}{2}\left(KU_f'(x_f[t])-q_{b(f),e(f)}[t]\right)^2 \leq B_4 < \infty, \quad (32)$$

for some B_4. To justify this claim, we ignore the $\alpha/2$ factor for now, and write

$$\begin{aligned}
&\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1}\sum_{f\in\mathcal{F}}\left(KU_f'(x_f[t])-q_{b(f),e(f)}[t]\right)^2\\
&= \limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1}\sum_{f\in\mathcal{F}}\Big[(KU_f'(x_f[t]))^2 \quad (33)\\
&\qquad -2KU_f'(x_f[t])q_{b(f),e(f)}[t]+q^2_{b(f),e(f)}[t]\Big]\\
&\overset{(a)}{\leq} K^2\sum_{f\in\mathcal{F}}(U_f'(m))^2+\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_{f\in\mathcal{F}}q^2_{b(f),e(f)}[t]\\
&\overset{(b)}{\leq} K^2\sum_{f\in\mathcal{F}}(U_f'(m))^2+c(\alpha,K,\tau) \quad (34)
\end{aligned}$$

where the inequality (a) is true since $x_f[t]\in[m,M]$, for all t and $f\in\mathcal{F}$ due to the nature of the primal-dual congestion controller, and since $U_f'(y)\geq 0$, for all $y\in[m,M]$. Also, inequality (b) follows from Proposition 1. Using the fact that $c(1/K^2,K,K) = O(K^2)$, (34) implies (32), which, when substituted into (31) shows that

$$\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\|\mathbf{x}[t]-\mathbf{x}^\star\|^2 \leq \frac{\tilde{B}^2}{K}, \quad (35)$$

where we have $\tilde{B}^2 := \frac{B_0+B_3+B_4}{\tilde{C}}$. Thus, for T large enough and for any $f\in\mathcal{F}$, we have

$$\begin{aligned}
\left|\frac{1}{T}\sum_{t=0}^{T-1}(x_f[t]-x_f^\star)\right| &\leq \frac{1}{T}\sum_{t=0}^{T-1}|x_f[t]-x_f^\star|\\
&\overset{(a)}{\leq} \sqrt{\frac{\tilde{B}^2}{K}} = \frac{\tilde{B}}{\sqrt{K}},
\end{aligned}$$

where inequality (a) follows from (35). ∎

Theorem 3 directly implies that the time-average rate allocation to the users can be made arbitrarily close to the optimal fair allocation by choosing K sufficiently large.

VI. Extensions and Variations

In this section, we discuss possible extensions and variations to the joint mechanism that we studied up to this point.

A. Stochastic Channel Models

To model channel variations, we assume that the network channel state can be in one of many states belonging to a finite set, say $\mathcal{J}$. Then, we let Γ_j denote the set of feasible link rates when the current state is $j\in\mathcal{J}$. Let π_j^s denote the stationary probability of the channel state being j. Then, we can define the average link capacity region as $\Gamma = \sum_{j\in\mathcal{J}}\pi_j^s\ \mathcal{CH}\{\Gamma_j\}$. Recall that $\mathcal{CH}\{A\}$ denotes the convex hull of the set A. Moreover, we define the average end-to-end capacity region Λ as in Definition 1. Then, the goal is to find flow rates so that $\sum_f U_f(x_f)$ is maximized over all the rates in Λ.

Assuming that the channel state is j at time t, the back-pressure policy performs the following optimization to determine the link rates:

$$\mu[t]\in\arg\max_{\{\eta\in\Gamma_j\}}\sum_{\{(n,m)\in\mathcal{L}\}}\eta_{(n,m)}w_{(n,m)}[t],$$

where $w_{(n,m)}[t]$ is defined as in (7). We can also allow randomness in the arrival process to model various implementation details. For example, the flow rates can be assumed to satisfy

$$\begin{aligned}
&E[x_f[t+1]\mid q_{b(f),e(f)}[t]]\\
&\quad=\{x_f[t]+\alpha(KU_f'(x_f[t])-q_{b(f),e(f)}[t])\}_m^M, \text{ and}\\
&\quad E[x_f^2[t]\mid q_{b(f),e(f)}[t]]\leq A<\infty, \qquad \forall q_{b(f),e(f)}[t]. \quad (36)
\end{aligned}$$

Under these modifications, a stochastic version of the stability and convergence results can be proven.

B. Dual Congestion Controller

A dual congestion controller is a gradient algorithm designed to minimize the dual objective of (4) (see [25] for the case of the Internet). If we allow randomness in the arrival process, the data rate $x_f[t]$ of flow f at time slot t is a random variable that satisfies (36) and

$$E[x_f[t]\mid q_{b(f),e(f)}[t]] = \min\left\{U_f'^{-1}\left(\frac{q_{b(f),e(f)}[t]}{K}\right),M\right\}.$$

The heuristic fluid model of this controller is given by

$$x_f(t) = U_f'^{-1}\left(\frac{q_{b(f),e(f)}(t)}{K}\right), \qquad \text{for all } f\in\mathcal{F}.$$

For this model, the global asymptotic stability of the queue lengths and the asymptotic optimality of the flow rates can again be proved using LaSalle's invariance principle by studying the Lyapunov function:

$$V(\mathbf{q};\lambda^\star) = \frac{1}{2}\sum_{n\in\mathcal{N}}\sum_{d\in\mathcal{N}}(\frac{q_{n,d}}{K}-\lambda^\star_{n,d})^2.$$

As in the case of the primal-dual algorithm, we can then establish the establish the stability and asymptotic optimality of the stochastic model described above. Our techniques here serve as an alternate proof of the results in [16], [26], [21].

C. Relationship to TCP

The primal-dual algorithm described here is similar in spirit to today's versions of TCP; however, we use queue lengths as the congestion feedback signal instead of packet loss which is the most common form of congestion signal in the Internet. Unlike the dual algorithm, the primal-dual algorithm adjusts the flow rates more gradually in response to network congestion.

VII. Conclusions

In this work, we propose and study a cross-layer resource allocation mechanism for wireless networks. It is shown that this algorithm achieves fairness and stability. Architecturally, we maintain the traditional protocol stack, but couple the layers through the use of queue-length information.

Appendix

A. The Dual Function (5):

By the definition of a dual function ([3]), we have
$D(\lambda) =$

$$\max_{\left\{\begin{array}{c}\mathbf{x}\geq\mathbf{0},\mu\in\Lambda,\\ \mu_{(n,m)}^{(d)}\geq 0,\forall d,\\ \mu_{(n,m)}=\sum_d \mu_{(n,m)}^{(d)}\end{array}\right\}} \left[\sum_{f\in\mathcal{F}} U_f(x_f) - \sum_{n,d}\left\{\lambda_{n,d} \times\left(\mu_{into(n)}^{(d)} + x_{n,d} - \mu_{out(n)}^{(d)}\right)\right\}\right].$$

where $x_{n,d} := \sum_f x_f \mathcal{I}_{\{b(f)=n,e(f)=d\}}$ denotes the total mean flow rate from n to d. The terms in the objective of this formulation can be re-ordered to get

$$\begin{aligned}
&\sum_{f\in\mathcal{F}}\left(U_f(x_f) - \lambda_{b(f),e(f)}x_f\right)\\
&-\sum_{n,d}\lambda_{n,d}(\mu_{into(n)}^{(d)} - \mu_{out(n)}^{(d)})\\
=\ &\sum_{f\in\mathcal{F}}\left(U_f(x_f) - \lambda_{b(f),e(f)}x_f\right)\\
&+\sum_{(n,m)\in\mathcal{L}}\sum_d \mu_{(n,m)}^{(d)}(\lambda_{n,d}-\lambda_{m,d}),
\end{aligned}$$

where the last step follows by manipulating the order of summations in the second term. Then, the maximization in the dual function can be decomposed into two parts as follows:
$D(\lambda) =$

$$\begin{aligned}
&\max_{\mathbf{x}\geq\mathbf{0}}\sum_{f\in\mathcal{F}}\left(U_f(x_f) - \lambda_{b(f),e(f)}x_f\right)\\
+&\max_{\left\{\begin{array}{c}\mu\in\Lambda,\mu_{(n,m)}^{(d)}\geq 0,\\ \mu_{(n,m)}=\sum_d \mu_{(n,m)}^{(d)}\end{array}\right\}}\sum_{(n,m)}\sum_d \mu_{(n,m)}^{(d)}(\lambda_{n,d}-\lambda_{m,d})\\
=&\sum_{f\in\mathcal{F}}\max_{x_f\geq 0}\left(U_f(x_f) - \lambda_{b(f),e(f)}x_f\right)\\
&+\max_{\mu\in\Lambda}\sum_{(n,m)}\mu_{(n,m)}\max_d(\lambda_{n,d}-\lambda_{m,d}),
\end{aligned}$$

where in the last step we used two facts: for the first maximization, the separability of the objective function together with the decoupled constraint set $\mathbf{x}\geq\mathbf{0}$ allow us to perform maximization over each term separately within the sum; for the second maximization, the linearity of the maximization implies that for each $(n,m)\in\mathcal{L}$, we will have $\mu_{(n,m)}^{d^*} = \mu_{(n,m)}$ where $d^* := \arg\max_d(\lambda_{n,d}-\lambda_{m,d})$, and $\mu_{(n,m)}^{d} = 0$ for $d\neq d^*$. ■

B. Proof of queue stability without using Assumption 1:

In the main body of the paper, we have shown two results for our primal-dual controller: that the average flow rates converges to the fair allocation; and that the entry queues $\{q_{b(f),e(f)}\}_f$ will be stable. These results were proved under the assumption that there is a flow between every source-destination pair in the network (Assumption 1). Under Assumption 1, the stability of the entry queues trivially implies the stability of all the queues. When this assumption is removed, the results on fairness continues to hold without modification provided that the stability result continues to hold without the Assumption 1.

To establish stability without using Assumption 1, we instead make the following reasonable assumptions:

Assumption 2: The set of feasible link rates satisfy the following assumptions:

(a) Λ is a discrete set.
(b) There exist μ_{min} satisfying $\mu_{min} := \min_{l\in\mathcal{L}}\min_{\mu:\mu_l>0}\mu_l$. (Thus, μ_{min} is the smallest non-zero rate that can be provided at any link.)
(c) Consider any link $l\in\mathcal{L}$. The set of link rates $\mu_l=\mu_{min}$ and $\mu_j = 0$, for all $j\neq l$, is feasible. (In other words, it is always feasible to choose any link's rate to be μ_{min} and choose all other rates to be zero.)

We will first consider the heuristic continuous-time fluid model.

Analysis of the Heuristic Continuous-time Model: By LaSalle's invariance principle, the system converges to a state satisfying $(20) = 0$. Therefore,

$$\sum_f x_f^\star q_{b(f),e(f)} \quad = \quad \sum_{n,d} q_{n,d}\left(\mu_{out(n)}^{(d)} - \mu_{into(n)}^{(d)}\right).$$

We also know that $\frac{q_{b(f),e(f)}}{K} = \lambda^\star_{b(f),e(f)}$. By rearranging the right-hand-side, we get

$$K\sum_f x_f^\star\lambda^\star_{b(f),e(f)} = \sum_{(n,m)}\sum_d \mu_{(n,m)}^{(d)}(q_{n,d}-q_{m,d})$$

This shows that the optimal value of the objective in the back-pressure algorithm (8) converges to a constant. This implies that the total queue-length is bounded (see equations (A.20)-(A.23) in [29]). To make the presentation self-contained, we present the argument in [29] below.

We assume that at least one path exists between any two nodes. Let (n_0,d_0) be such that $q_{n_0,d_0} = \max_{n,d} q_{n,d}$. In other words, q_{n_0,d_0} is the largest queue in the network. Let n_0 and d_0 be connected by a path through the nodes $n_1,\cdots,n_J$.

Then, we can write

$$\begin{aligned} q_{n_0,d_0} &= \sum_{i=0}^{J-1}(q_{n_i,d_0} - q_{n_{i+1},d_0}) \\ &\leq J \max_i \max_d (q_{n_i,d} - q_{n_{i+1},d}) \\ &\leq J \sum_{(n,m)} \frac{\mu_{(n,m)}}{\mu_{min}} \max_d (q_{n,d} - q_{m,d}), \end{aligned}$$

where the last inequality can be established by contradiction: if the inequality were not true, we could have assigned μ_{min} to the link with the maximum differential backlog on the path from n_0 to d_0 and zero to the rest of the links in the network, and obtained a larger value of the back-pressure objective. Thus,

$$\begin{aligned} \sum_{n,d} q_{n,d} &\leq \frac{J|\mathcal{N}|^2}{\mu_{min}} \sum_{(n,m)} \mu_{(n,m)} \max_d (q_{n,d} - q_{m,d}) \quad (37) \\ &= K \sum_f x_f^\star \lambda_{b(f),e(f)}^\star < \infty \end{aligned}$$

and our result is proved. ■.

Analysis of the Discrete-time System: When convenient, we will omit the time index $[t]$ unless there is ambiguity. Consider the drift of the Lyapunov function $L(\mathbf{q}) = \frac{1}{2}\sum_{n,d} q_{n,d}^2$.

$$\begin{aligned} \Delta L_t(\mathbf{q}) &= \frac{1}{2}\sum_{n,d} q_{n,d}^2[t+1] - \frac{1}{2}\sum_{n,d} q_{n,d}^2[t] \\ &= \frac{1}{2}\sum_{n,d}\left(q_{n,d} + x_{n,d} + s_{into(n)}^{(d)} - s_{out(n)}^{(d)}\right)^2 \\ &\quad -\frac{1}{2}\sum_{n,d} q_{n,d}^2, \end{aligned}$$

where $x_{n,d} := \sum_{f:b(f)=n,e(f)=d} x_f$. Since the link rates and external arrival rates are bounded, we can upper-bound the previous expression as

$$\Delta L_t(\mathbf{q}) \leq \sum_{n,d} q_{n,d}\left(x_{n,d} + s_{into(n)}^{(d)} - s_{out(n)}^{(d)}\right) + \hat{B}_1,$$

for some bounded $\hat{B}_1$. Then, using Lemma 1, we can further write

$$\begin{aligned} \Delta L_t(\mathbf{q}) &\leq \sum_{n,d} q_{n,d}\left(x_{n,d} + \mu_{into(n)}^{(d)} - \mu_{out(n)}^{(d)}\right) + \hat{B}_1 + B \\ &\leq \sum_f q_{b(f),e(f)} x_f - \sum_{(n,m)} \mu_{(n,m)} \max_d (q_{n,d} - q_{m,d}) \\ &\quad +\hat{B}_1 + B \end{aligned}$$

Next, we use the bound (37) to write

$$\Delta L_t(\mathbf{q}) \leq \sum_f q_{b(f),e(f)} x_f - \theta \sum_{n,d} q_{n,d} + \hat{B}_1 + B,$$

for some $\theta > 0$.

Recall from the proof of Proposition 1 that, when $q_{b(f),e(f)} \geq g(\alpha, K)$, then $x_f \leq m$. Therefore, we can write

$$\begin{aligned} \Delta L_t(\mathbf{q}) &\leq m\sum_f q_{b(f),e(f)} \mathcal{I}_{q_{b(f),e(f)} \geq g(\alpha,K)} - \theta \sum_{n,d} q_{n,d} \\ &\quad +\hat{B}_2(\alpha,K) + \hat{B}_1 + B \\ &\leq m\sum_f q_{b(f),e(f)} - \theta\sum_{n,d} q_{n,d} \\ &\quad +\hat{B}_3(\alpha,K) + \hat{B}_2(\alpha,K) + \hat{B}_1 + B, \end{aligned}$$

where $\hat{B}_2(\alpha, K)$ and $\hat{B}_3(\alpha, K)$ are bounded terms. Note that $\sum_f q_{b(f),e(f)}$ may count some ingress queues more than once if there is more than one flow between some source-destination pairs. Let Z denote the maximum number of flows that share a source-destination pair. Then, we can write

$$\begin{aligned} \Delta L_t(\mathbf{q}) &\leq mZ\sum_{n,d} q_{n,d} - \theta\sum_{n,d} q_{n,d} \\ &\quad +\hat{B}_3(\alpha,K) + \hat{B}_2(\alpha,K) + \hat{B}_1 + B \\ &= (mZ - \theta)\sum_{n,d} q_{n,d} + \hat{B}_3(\alpha,K) + \hat{B}_2(\alpha,K) \\ &\quad +\hat{B}_1 + B. \end{aligned}$$

By choosing m sufficiently small, we can assure that $(mZ - \theta) \leq -\epsilon$ for some $\epsilon > 0$. As in Proposition 1, the stability of the network follows. ■

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