Correction to "Exploiting Channel Memory for Joint Estimation and Scheduling in Downlink Networks - a Whittles Indexability Analysis"

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In the above paper [1], in Proposition 2, case (iii), it was argued that since reward functions $V_\omega^0(\pi)$ and $V_\omega^1(\pi)$ are convex with inequality orders reversed at the ends of the support space: $\pi \in [0,1]$, they must intersect only once. In general, however, we may carefully construct pairs of convex functions such as $(x^2, x^2 - \sin(x))$ that intersect multiple times.

In this addendum, we address this and make rigorous the proof of Proposition 2, case (iii) for a certain class of scheduling system parameters and conjecture that the uniqueness of intersection holds for general cases as well.

I. Preliminaries

The reward functions in the Whittle's indexability framework are recalled from [1] first. Total reward upon 'idle' action in current slot and optimal actions in future slots is given by,

$$V_\omega^0(\pi) = \omega + \beta V_\omega(Q(\pi)).$$

Total reward upon 'transmit' action in current slot and optimal actions in future slots is given by,

$$V_\omega^1(\pi) = R(\pi) + \beta[\pi V_\omega(p) + (1-\pi)V_\omega(r)],$$

where, recall from [1] that, $Q(\pi) = \pi(p-r) + r$ is the belief-evolution function; $R(\pi)$ is the immediate reward; p, r are Markov channel parameters; ω is the subsidy for idle decision and β is the discount factor.

II. Assumptions

We consider a class of scheduling system parameters that satisfy the following assumptions.

1. The channels are positively correlated, i.e., $p_i > r_i$ for each user i in the original multi-user scheduling problem. For ease of exposition, we drop the subscript i in the following.
2. Immediate reward $R(\pi)$ has the following structural properties. For any π_1, π_2 such that $0 \le \pi_1 < \pi_2 \le 1$,
   a. $R(\pi_2) > R(\pi_1)$, i.e., $R(\pi)$ strictly increases in π.
   b. $R(\pi_2) - R(\pi_1) > \beta(R(Q(\pi_2)) - R(Q(\pi_1)))$. This is *contraction mapping* with $Q(\pi) = (p-r)\pi + r$ being the contraction or contractor on metric space $\pi \in [0,1]$, with distance measure $d(\pi_2, \pi_1) = |R(\pi_2) - R(\pi_1)|$.

Comments on the Assumptions:

We now discuss the implications and prevalence of scheduling systems that satisfy Assumption 1-2.

Assumption 1: This covers a large class of fading channels where channel condition can be expected to evolve in a smooth fashion across time-slots. Assumption 2a: Note that from Lemma 1(a) in [1], $R(\pi)$ is already proven to be an increasing function of π. We have added the strict monotonicity in this assumption. This is also intuitive and expected to cover a large class of estimator - rate adapter pairs, as any increase in belief, π, can be expected to translate to a non-zero increase in the immediate reward. Assumption 2b: $R(\pi)$ is established to be convex in Lemma 1(a) in[1]. Recall that π^0 denotes the steady state probability of being in state h. Thus for $0 \le \pi^0 < \pi_1 < \pi_2 \le 1$, it is directly shown that $R(\pi_1) - R(\pi_2) > \beta(R(Q(\pi_1)) - R(Q(\pi_2)))$, since $\pi_1 - \pi_2 > Q(\pi_1) - Q(\pi_2)$, $\pi_2 > Q(\pi_2)$, $\pi_1 > Q(\pi_1)$. The assumption covers the remaining pairs of (π_1, π_2) , thereby imposing a contraction mapping on a measure of $R(\pi)$.

Existence of Estimator - Rate Adapter Pairs:

We will now demonstrate that there exists estimator - rate adapter pairs that satisfy Assumption 2b. We proceed to construct one such estimator - rate adapter pair. Note from Lemma 1 in [1] that, $R(\pi)$ is a point-wise maximum over a family of linear functions, each of which represent the immediate reward of a unique estimator-rate adapter pair. Construct an cumulative estimator - rate adapter pair $U_c(\pi)$ such that

$$U_c(\pi) = \begin{cases} u_0(\pi), & \text{if } \pi \in [0, \pi^0]. \\ u^*(\pi), & \text{if } \pi \in (\pi^0, 1]. \end{cases} \tag{1}$$

where π^0 is the steady state probability of channel being in high-state. $u_0(\pi)$ is a unique estimator - rate adapter pair that is linear and monotonically increasing in π. This could be chosen with an objective such as: maximize immediate reward for π close to 0. Further, $u^*(\pi)$ is the *optimal* estimator - rate adapter pair at π. Now consider the following 3 cases for the pair (π_1, π_2).

- Case 1. $0 \le \pi_1 < \pi_2 \le \pi^0$: Since $U_c(\pi) = u_0(\pi)$ in this range of π, we have $U_c(\pi)$ is linear and strictly increasing in π within which contraction mapping in Assumption 2b strictly holds.
- Case 2. $0 \le \pi_1 \le \pi^0 < \pi_2$: It is easily shown that $Q(\pi_1) \in [\pi_1, \pi^0]$ and $Q(\pi_2) \in (\pi^0, \pi_2]$. Along with the fact that $R(\pi)$ is strictly increasing in π, ontraction mapping in Assumption 2b is established.
- Case 3. $0 \le \pi^0 \le \pi_1 < \pi_2 \le 1$: As noted within Assumption 2b, the contraction mapping readily holds for this case using Lemma 1a in [1].

This demonstrates the existence of estimator - rate adapter pairs that satisfy Assumption 2b.

We now proceed with the proof.

III. Claim

Reward functions $V_\omega^0(\pi)$ and $V_\omega^1(\pi)$ intersect at most once in the region $\pi \in [0, 1]$ under Assumptions 1 and 2.

Proof Approach:

We prove the claim by contradiction. Suppose there are multiple intersections, denoted as $\pi_1, \pi_2, \cdots, \pi_n$ with $0 \le \pi_1 < \pi_2 < \cdots < \pi_n \le 1$ and $n \ge 3$, we prove the Claim by considering the following four exhaustive cases based on steady state probability, π^0. Note that if there are more than one intersections, there must be at least three intersections since the relationship of $V_\omega^0(\pi)$ and $V_\omega^1(\pi)$ is reversed at the end points 0 and 1 as established in Proposition 2 in [1].

- Case 1: The value of π^0 is less than all intersections, i.e., $0 \le \pi^0 < \pi_1$.
- Case 2: The value of $\pi_1 \le \pi^0 < \pi_n$, and π^0 is within active region, i.e., $V_\omega^1(\pi^0) > V_\omega^0(\pi^0)$ if $\pi^0 \notin \{\pi_1, \pi_2, \cdots, \pi_n\}$; $V_\omega^1(\pi^0) = V_\omega^0(\pi^0)$ if $\pi^0 \in \{\pi_1, \pi_2, \cdots, \pi_n\}$
- Case 3: The value of $\pi_1 \le \pi^0 < \pi_n$, and π^0 is within idle region, i.e., $V_\omega^1(\pi^0) < V_\omega^0(\pi^0)$ if $\pi^0 \notin \{\pi_1, \pi_2, \cdots, \pi_n\}$; $V_\omega^1(\pi^0) = V_\omega^0(\pi^0)$ if $\pi^0 \in \{\pi_1, \pi_2, \cdots, \pi_n\}$
- Case 4: The value of π^0 is greater than all intersections, i.e., $\pi^0 \ge \pi_n$.

First, we establish the following structural property of reward functions.

Lemma 1.

$$V_\omega(\pi_a) \ge V_\omega(\pi_b) \ \forall \ \pi_a > \pi_b$$
$$V_\omega^1(\pi_a) > V_\omega^1(\pi_b) \ \forall \ \pi_a > \pi_b$$

Proof. Similar to the proof of Proposition 1 in [1], we let $\widetilde{V}_{\omega,t}(\pi)$ be the optimal reward function at time t for M-stage finite horizon problem. Similarly, let $\widehat{V}_{\omega,t}^1(\pi)$ (or $\widehat{V}_{\omega,t}^0(\pi)$) be the reward function upon transmit (or idle) and then optimal decisions for the M-stage finite horizon problem, and let $\widehat{V}_{\omega,t}^1(\pi)$ be the corresponding reward at time t.

Then at time M, the Lemma holds since $\widetilde{V}_{\omega,M}(\pi_a) = R(\pi_a) > R(\pi_b) = \widetilde{V}_{\omega,M}(\pi_b)$. Similarly, $\widehat{V}_{\omega,M}^1(\pi_a) = R(\pi_a) > R(\pi_b) = \widehat{V}_{\omega,M}^1(\pi_b)$. Here $R(\pi_a) > R(\pi_b)$ follows from Assumption 2a.

Suppose at time t, $\widetilde{V}_{\omega,t}(\pi_a) \ge \widetilde{V}_{\omega,t}(\pi_b)$ and $\widehat{V}_{\omega,t}^1(\pi_a) > \widehat{V}_{\omega,t}^1(\pi_b)$.

Then at time $t-1$, we have $\widetilde{V}_{\omega,t-1}(\pi) = \max\{\widehat{V}_{\omega,t-1}^0(\pi), \widehat{V}_{\omega,t-1}^1(\pi)\}$, where

$$\begin{aligned}
\widehat{V}_{\omega,t-1}^0(\pi) &= \omega + \beta \widetilde{V}_{\omega,t}(p\pi + (1-\pi)r) \\
\widehat{V}_{\omega,t-1}^1(\pi) &= R(\pi) + \beta \cdot [\pi \widetilde{V}_{\omega,t}(p) + (1-\pi)\widehat{V}_{\omega,t}(r)] \\
&= R(\pi) + \beta \cdot [\pi[\widetilde{V}_{\omega,t}(p) - \widetilde{V}_{\omega,t}(r)] + \widetilde{V}_{\omega,t}(r)]
\end{aligned}$$

Note that since $(p-r)\pi$ increases with π and $\widetilde{V}_{\omega,t}(\pi)$ increases with π (induction), we have $\widehat{V}_{\omega,t-1}^0(\pi)$ increases with π.

Since $R(\pi)$ strictly increases with π (from Assumption 2a) and $\pi[\widetilde{V}_{\omega,t}(p) - \widetilde{V}_{\omega,t}(r)]$ increases with π (induction), we have $\widehat{V}_{\omega,t-1}^1(\pi)$ strictly increases with π.

Therefore $\widetilde{V}_{\omega,t-1}(\pi)$ increases with π as maximum of two increasing functions of π. Using induction on $\widehat{V}_{\omega,t}^1(\pi)$ and $\widetilde{V}_{\omega,t}(\pi)$, the lemma is thus established. ∎

Recall from proof of Proposition 1 in [1], the following relation between $V_\omega^0(\pi)$ and $V_\omega^1(\pi)$ at extremes of belief values:

$$V_\omega^0(0) > V_\omega^1(0)$$
$$V_\omega^0(1) < V_\omega^1(1) \tag{2}$$

Thus, with $\pi_1, \pi_2, \cdots, \pi_n$ indicating the multiple intersections, we have

$$V_\omega^0(\pi) > V_\omega^1(\pi), \forall \pi \in [0, \pi_1)$$
$$V_\omega^0(\pi) < V_\omega^1(\pi), \forall \pi \in (\pi_n, 1]. \tag{3}$$

IV. Case 1

In this case, all the intersections of $V_\omega^0(\pi)$ and $V_\omega^1(\pi)$ are greater than π^0. We then have $\pi^0 < \pi_1 < \pi_2 < \cdots$.

Note that at the first intersection π_1 we have $V_\omega^0(\pi) = V_\omega^1(\pi)$ and

$$V_\omega^0(\pi_1) = \omega + \beta\omega + \beta^2\omega + \cdots = \frac{\omega}{1-\beta} \tag{4}$$
$$V_\omega^1(\pi_1) = R(\pi_1) + \beta \cdot [\pi_1 V_\omega(p) + (1-\pi_1)V_\omega(r)], \tag{5}$$

where the expression of $V_\omega^0(\pi_1)$ holds because if it is optimal to stay idle at π_1 at one slot, then it will be optimal to stay idle forever since $Q^k(\pi_1) < \pi_1$ and from (3) it is also in idle region for $k \geq 1$.

At the second intersection π_2, we discuss the following two sub-cases.

(Case 1.1). $Q(\pi_2)$ is within idle region. Then we have $V_\omega^0(\pi_2) = V_\omega^1(\pi_2)$ and

$$V_\omega^0(\pi_2) = \frac{\omega}{1-\beta}, \tag{6}$$
$$V_\omega^1(\pi_2) = R(\pi_2) + \beta \cdot [\pi_2 V_\omega(p) + (1-\pi_2)V_\omega(r)]. \tag{7}$$

where the expression of $V_\omega^0(\pi_2)$ holds because if $Q(\pi_2)$ is in idle region, then $Q^k(\pi_2)$ is also in idle region for $k \geq 0$.

From (4) and (6)we have $V_\omega^0(\pi_1) = V_\omega^0(\pi_2)$. Since both π_1 and π_2 are at the intersection of $V_\omega^0(\pi)$ and $V_\omega^1(\pi)$, we have $V_\omega^0(\pi_1) = V_\omega^1(\pi_1)$ and $V_\omega^0(\pi_2) = V_\omega^1(\pi_2)$. We hence have $V_\omega^1(\pi_1) = V_\omega^1(\pi_2)$. This contradicts the result of Lemma 1 that $V_\omega^1(\pi)$ strictly increases with π. Thus this case is not feasible.

(Case 1.2.) $Q(\pi_2)$ is within active region. Then there must exist another π_3 such that $Q(\pi_3)$ is within the active region as well and $\pi_1 < \pi_3 < \pi_2$. Therefore

$$V_\omega^0(\pi_2) = \omega + \beta \cdot V_\omega^1(Q(\pi_2)) \tag{8}$$
$$V_\omega^1(\pi_2) = R(\pi_2) + \beta \cdot [\pi_2 V_\omega(p) + (1-\pi_2)V_\omega(r)], \tag{9}$$

with $V_\omega^0(\pi_2) = V_\omega^1(\pi_2)$. Also, at π_3 we have

$$V_\omega^0(\pi_3) = \omega + \beta \cdot V_\omega^1(Q(\pi_3)) \tag{10}$$
$$V_\omega^1(\pi_3) = R(\pi_3) + \beta \cdot [\pi_3 V_\omega(p) + (1-\pi_3)V_\omega(r)], \tag{11}$$

with $V_\omega^0(\pi_3) < V_\omega^1(\pi_3)$ since π_3 is in active region. From (9) and (11)

$$V_\omega^1(\pi_2) - V_\omega^1(\pi_3) = R(\pi_2) - R(\pi_3) + \beta[(\pi_2 - \pi_3)(V_\omega(p) - V_\omega(r))]. \tag{12}$$

Also from (8) and (10) we have

$$\begin{aligned} V_\omega^0(\pi_2) - V_\omega^0(\pi_3) =& \beta V_\omega^1(Q(\pi_2)) - \beta V_\omega^1(Q(\pi_3)) \\ =& \beta[R(Q(\pi_2)) - R(Q(\pi_3))] + \beta[Q(\pi_2) - Q(\pi_3)][V_\omega(p) - V_\omega(r)], \end{aligned} \tag{13}$$

Since $\pi_2 - \pi_3 > Q(\pi_2) - Q(\pi_3) = (p-r)(\pi_2 - \pi_3)$ and $R(\pi_2) - R(\pi_3) > R(Q(\pi_2)) - R(Q(\pi_3))$, from (12) and (13) we have $V_\omega^1(\pi_2) - V_\omega^1(\pi_3) > V_\omega^0(\pi_2) - V_\omega^0(\pi_3)$. Therefore $V_\omega^1(\pi_3) - V_\omega^0(\pi_3) < V_\omega^1(\pi_2) - V_\omega^0(\pi_2) = 0$. Thus $V_\omega^1(\pi_3) < V_\omega^0(\pi_3)$. This contradicts the fact that π_3 belongs to the active region. Thus this case is not feasible.

V. Case 2

Suppose π^0 is within active region and $\pi_k \leq \pi^0 < \pi_{k+1}$ for some $1 \leq k < n$ where $V_\omega^1(\pi^0) > V_\omega^0(\pi^0)$. Next consider $\tilde{\pi}$ such that $\pi_{k+1} < \tilde{\pi} < \pi_{k+2}$, i.e., $\tilde{\pi}$ is in the immediate idle interval greater than π^0. Note that $\exists\ \tilde{\pi}$ such that $\pi^0 < Q(\tilde{\pi}) < \pi_{k+1}$. Thus $Q(\tilde{\pi})$ is in active region. We hence have

$$V_\omega^1(\tilde{\pi}) = R(\tilde{\pi}) + \beta \cdot [\tilde{\pi} V_\omega(p) + (1-\tilde{\pi})V_\omega(r)] \tag{14}$$
$$V_\omega^0(\tilde{\pi}) = \omega + \beta\big[R(Q(\tilde{\pi})) + \beta[Q(\tilde{\pi})V_\omega(p) + \big(1 - Q(\tilde{\pi})\big)V_\omega(r)]\big] \tag{15}$$

We present the following lemma.

Lemma 2.

$$\begin{aligned}&\beta\tilde{\pi}[V_\omega(p)-V_\omega(r)]-\beta^2[Q(\tilde{\pi})\cdot[V_\omega(p)-V_\omega(r)]]\\ \le&\beta\pi^0[V_\omega(p)-V_\omega(r)]-\beta^2\pi^0[V_\omega(p)-V_\omega(r)].\end{aligned}$$

Proof. Rearranging terms we have

$$\beta[V_\omega(p)-V_\omega(r)][\tilde{\pi}-\pi^0]\ge\beta^2[V_\omega(p)-V_\omega(r)][Q(\tilde{\pi})-\pi^0],$$

which holds since $V_\omega(p)\ge V_\omega(r)$ and $\tilde{\pi}>Q(\tilde{\pi})$. □

From (14) and (15) we have,

$$\begin{aligned}&V_\omega^1(\tilde{\pi})-V_\omega^0(\tilde{\pi})\\ =&R(\tilde{\pi})+\beta\cdot[\tilde{\pi}V_\omega(p)+(1-\tilde{\pi})V_\omega(r)]\\ &\quad-\Big[\omega+\beta\big[R(Q(\tilde{\pi}))+\beta\cdot[R(Q(\tilde{\pi}))V_\omega(p)+\big(1-R(Q(\tilde{\pi}))\big)V_\omega(r)]\big]\Big]\\ =&R(\tilde{\pi})-\beta R(Q(\tilde{\pi}))+\beta[V_\omega(p)-V_\omega(r)](\tilde{\pi}-\beta Q(\tilde{\pi}))+\beta V_\omega(r)-\omega-\beta^2V_\omega(r)\\ >&R(\tilde{\pi})-\beta R(\tilde{\pi})+\beta[V_\omega(p)-V_\omega(r)](\tilde{\pi}-\beta Q(\tilde{\pi}))+\beta V_\omega(r)-\omega-\beta^2V_\omega(r)\\ >&R(\pi^0)-\beta R(\pi^0)+\beta[V_\omega(p)-V_\omega(r)](\pi^0-\beta Q(\pi^0))+\beta V_\omega(r)-\omega-\beta^2V_\omega(r)\\ =&R(\pi^0)+\beta[\pi^0V_\omega(p)-(1-\pi^0)V_\omega(r)]-\Big[\omega+\beta\big[R(\pi^0)+\beta[\pi^0V_\omega(p)+(1-\pi^0)V_\omega(r)]\big]\Big]\\ =&V_\omega^1(\pi^0)-V_\omega^0(\pi^0)\\ \ge&0,\end{aligned}$$

where the first inequality holds since $Q(\tilde{\pi})<\tilde{\pi}$ and hence $R(Q(\tilde{\pi}))<R(\tilde{\pi})$ from Lemma 1. The second inequality holds because $\tilde{\pi}>\pi^0$ and hence $R(\tilde{\pi})>R(\pi^0)$ and $\tilde{\pi}-\beta Q(\tilde{\pi})>\pi^0-\beta Q(\pi^0)$. The last inequality holds because π^0 is within the active region. In fact, if $\pi^0>\pi_k$, we have $V_\omega^1(\tilde{\pi})-V_\omega^0(\tilde{\pi})>0$ and if $\pi^0=\pi_k$, we have $V_\omega^1(\tilde{\pi})-V_\omega^0(\tilde{\pi})=0$. The above expressions contradict with the assumption that $\tilde{\pi}$ is strictly within idle region, i.e., $V_\omega^1(\tilde{\pi})<V_\omega^0(\tilde{\pi})$. This contradiction makes this case infeasible.

VI. CASE 3

Suppose $\pi_k\le\pi^0<\pi_{k+1}$, $k\ge1$, and π^0 is within idle region. Note that for all belief values π in the interval $[\pi_k,\pi_{k+1}]$, we have

$$V_\omega^0(\pi)=\omega+\beta\omega+\beta^2\omega+\cdots=\frac{\omega}{1-\beta},\tag{16}$$

since $Q^k(\pi)$ is in idle region.

In contrast, from Lemma 1, $V_\omega^1(\pi)$ strictly increases in that region. We hence have $V_\omega^1(\pi_{k+1})>V_\omega^1(\pi_k)$. Note that at π_k and π_{k+1}, we have $V_\omega^0(\pi_k)=V_\omega^1(\pi_{k+1})$ and $V_\omega^1(\pi_k)=V_\omega^0(\pi_k)$. Therefore $V_\omega^1(\pi_{k+1})=V_\omega^1(\pi_k)$, which contradicts $V_\omega^1(\pi_{k+1})>V_\omega^1(\pi_k)$. This contraction makes this case infeasible.

VII. CASE 4

We suppose π^0 is to the right of all intersections, i.e., $\pi^0\ge\pi_n>\pi_{n-1}>\cdots$. Therefore we have

$$V_\omega^0(\pi_n)=\omega+\beta\big[R(\pi_n)+\beta[Q(\pi_n)V_\omega(p)+(1-Q(\pi_n))V_\omega(r)]\big]\tag{17}$$

$$V_\omega^1(\pi_n)=R(\pi_n)+\beta\cdot[\pi_nV_\omega(p)+(1-\pi_n)V_\omega(r)],\tag{18}$$

where 17 holds since $Q(\pi_n)>\pi_n$ and from (3) it is in active region. Since $V_\omega^0(\pi_n)=V_\omega^1(\pi_n)$, we have

$$\omega=R(\pi_n)-\beta R(Q(\pi_n))+\beta[V_\omega(p)-V_\omega(r)](\pi_n-\beta Q(\pi_n))+\beta(1-\beta)V_\omega(r)\tag{19}$$

Consider $\hat{\pi}\in(\pi_{n-2},\pi_{n-1})$, i.e., $\hat{\pi}$ is in an active region. We have

$$\begin{aligned}V_\omega^1(\hat{\pi})&=R(\hat{\pi})+\beta\cdot[\hat{\pi}V_\omega(p)+(1-\hat{\pi})V_\omega(r)],\\ V_\omega^0(\hat{\pi})&=\omega+\beta V_\omega(Q(\hat{\pi}))\\ &\ge\omega+\beta\big[R(Q(\hat{\pi}))+\beta[Q(\hat{\pi})V_\omega(p)+(1-Q(\hat{\pi}))V_\omega(r)]\big]\end{aligned}$$

where the last inequality is because the reward obtained by idle followed by optimal decisions is better than the reward obtained by idle followed by an active decision.

Since $V_\omega^1(\hat{\pi}) > V_\omega^0(\hat{\pi})$, we have

$$\begin{aligned}\omega &< R(\hat{\pi}) - \beta R(Q(\hat{\pi})) + \beta[V_\omega(p) - V_\omega(r)](\hat{\pi} - \beta Q(\hat{\pi})) + \beta(1-\beta)V_\omega(r) \\ &< R(\hat{\pi}) - \beta R(Q(\hat{\pi})) + \beta[V_\omega(p) - V_\omega(r)](\pi_n - \beta Q(\pi_n)) + \beta(1-\beta)V_\omega(r) \\ &< R(\pi_n) - \beta R(Q(\pi_n)) + \beta[V_\omega(p) - V_\omega(r)](\pi_n - \beta Q(\pi_n)) + \beta(1-\beta)V_\omega(r)\end{aligned} \tag{20}$$

where the second inequality comes from

$$\pi_n - \beta Q(\pi_n) > \hat{\pi} - \beta Q(\hat{\pi}), \tag{21}$$

since, with $\pi_n > \hat{\pi}$,

$$[\pi_n - \beta Q(\pi_n)] - [\hat{\pi} - \beta Q(\hat{\pi})] = (1 - (p - r))(\pi_n - \hat{\pi}) > 0.$$

The last inequality in (20) uses Assumption 2b: $R(\pi_n) - R(\hat{\pi}) > \beta R(Q(\pi_n)) - \beta R(Q(\hat{\pi}))$. Considering (20), we have

$$\begin{aligned}\omega &< R(\pi_n) - \beta R(Q(\pi_n)) + \beta[V_\omega(p) - V_\omega(r)](\pi_n - \beta Q(\pi_n)) + \beta(1-\beta)V_\omega(r) \\ &= \omega\end{aligned}$$

where the last equality follows from (19). Thus we have the contradiction $\omega > \omega$, making this case infeasible.

VIII. Acknowledgement

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References

[1] Ouyang Wenzhuo, Sugumar Murugesan, Atilla Eryilmaz, and Ness B. Shroff, "Exploiting Channel Memory for Joint Estimation and Scheduling in Downlink Networksa Whittles Indexability Analysis." *IEEE Transactions on Information Theory*, vol. 61, no. 4, pp. 1702-1719, 2005.